6.3 Estimate Square Roots

Common Core Standards
8. NS.1 Know that there are numbers that are not rational, and approximate them by rational numbers. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

8. NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π 2). For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.
WARM-UP

Evaluate the square roots of the perfect squares.

\[
\begin{array}{ll}
\sqrt{1} & \sqrt{81} \\
\sqrt{4} & \sqrt{100} \\
\sqrt{9} & \sqrt{121} \\
\sqrt{16} & \sqrt{144} \\
\sqrt{25} & \sqrt{169} \\
\sqrt{36} & \sqrt{196} \\
\sqrt{49} & \sqrt{225} \\
\end{array}
\]
Estimate Square Roots

Are there answers to other square roots?

\[ \sqrt{2} \quad \sqrt{3} \quad \sqrt{5} \quad \sqrt{6} \quad \sqrt{7} \quad \sqrt{8} \quad \sqrt{10} \quad \sqrt{11} \]
There are three kinds of decimals. **Terminating** – they end. **Repeating** – they repeat the same digits forever. **Irrational** – they don’t terminate or repeat.

**Concept Check**
Describe each as terminating, repeating, or irrational.

\[
\begin{array}{c|c}
.5 & .555... \\
3.1415926... & \\
\end{array}
\]
NOTES

There are only a few perfect squares. The square roots of the other whole numbers are **irrational**.

\[
\sqrt{5} = 2.236067977... \\
\sqrt{23} = 4.795831523...
\]

**Concept Check**

Which is **not** a perfect square?

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<td>108</td>
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Irrational square roots always fall between the square roots of the perfect squares they are between.

\[ \sqrt{1} \quad \sqrt{4} \quad \sqrt{9} \quad \sqrt{16} \quad \sqrt{25} \quad \sqrt{36} \quad \sqrt{49} \quad \sqrt{64} \quad \sqrt{81} \quad \sqrt{100} \quad \sqrt{121} \quad \sqrt{144} \]

**Examples**

The positive square root of 7 falls between what two whole numbers?

The positive square root of 19 falls between what two whole numbers?
EXAMPLES

Find the two whole numbers the square root falls between.

\[ \sqrt{1} \quad \sqrt{4} \quad \sqrt{9} \quad \sqrt{16} \quad \sqrt{25} \quad \sqrt{36} \quad \sqrt{49} \quad \sqrt{64} \quad \sqrt{81} \quad \sqrt{100} \quad \sqrt{121} \quad \sqrt{144} \]

\[ \sqrt{41} \quad \sqrt{67} \]

\[ \sqrt{103} \]
EXAMPLES

Find the two whole numbers the square root falls between.

$\sqrt{170}$  $\sqrt{224}$
EXAMPLES

Round the square root to the nearest whole number.

\[ \sqrt{27} \quad \sqrt{61} \]
Which is **not** a perfect square?

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<th>1</th>
<th>9</th>
<th>15</th>
<th>16</th>
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Which is **not** a perfect square?

<table>
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<th>36</th>
<th>48</th>
<th>49</th>
<th>64</th>
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</table>

Find the two whole numbers the square root falls between.

\[ \sqrt{2} \]  \hspace{2cm} \sqrt{61} \]

| \sqrt{32} | \sqrt{119} |
PRACTICE

Round the square root to the nearest whole number.

\[ \sqrt{45} \]
FINAL QUESTION

Find the two whole numbers the square root falls between.

\[ \sqrt{407} \]