6.5 Irrational Versus Rational Numbers

Common Core Standards
8. NS.1 Know that there are numbers that are not rational, and approximate them by rational numbers. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
8. NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., \( \pi^2 \)). For example, by truncating the decimal expansion of \( \sqrt{2} \), show that \( \sqrt{2} \) is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.
WARM-UP

Evaluate the perfect square root.

1) \(\sqrt{9}\)  
2) \(\sqrt{16}\)

3) Round the square root to the nearest tenth.

\(\sqrt{11}\)

4) Accurately position \(\sqrt{11}\) on the number line.

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[Number line diagram with points marked at 3, 4, 5, 6, and 7]
Irrational Versus Rational Numbers

If numbers that never terminate or repeat are called irrational, what are all the other numbers called?

\[
\begin{align*}
\frac{1}{3} & \quad .2 & \quad 7,000 & \quad 2.4 \times 10^{-2} \\
.333... & \quad .3 & \quad -32 \frac{1}{2} & \quad -25 & \quad 9^2 \\
\sqrt{11} & \quad 3.1415926... & \quad \pi & \quad 1.7320508... \\
\end{align*}
\]
NOTES

There are two types of real numbers.

**Rational** – Positive and negative numbers, fractions, and terminating and repeating decimals.

**Irrational** – Decimals that are non-terminating and non-repeating (3.1415926…).

**Concept Check**

Copy down only the irrational number.

\[
3.8729..., \quad \bar{3}, \quad .825, \quad \frac{1}{3}, \quad \frac{5}{2}, \quad 5.3851..., \quad -4\frac{1}{6}, \quad -2.236..., \quad -2.111...
\]
NOTES

The square roots of whole numbers that are not perfect squares are all irrational. That’s why we have to estimate them. \( \sqrt{2} = 1.414213562... \) irrational

Examples

\( \sqrt{9} = 3 \) rational

Copy the irrational number.

\[ \sqrt{4} \quad \sqrt{5} \quad \sqrt{25} \quad \sqrt{64} \]

\[ \sqrt{9} \quad \sqrt{16} \quad \sqrt{36} \quad \sqrt{50} \]
EXAMPLES

Copy the irrational number.

\[ .12 \quad .45 \quad \sqrt{8} \quad \sqrt{36} \]

\[ 1 \quad -3 \frac{5}{6} \quad \sqrt{42} \quad \sqrt{81} \]

\[ .\overline{4} \quad \sqrt{81} \quad 4.123105\ldots \quad \sqrt{121} \]

\[ .8\overline{3} \quad 4.242 \quad \sqrt{2} \quad .\overline{333}\ldots \]
If an expression contains any irrational number, the entire expression is probably irrational.

Examples
Copy the expression that is irrational.

\[
\begin{align*}
\frac{\sqrt{4}}{2} & \quad \frac{\sqrt{5}}{2} & \quad \frac{\sqrt{9}}{2} & \quad \frac{\sqrt{36}}{2} \\
3\sqrt{7} & \quad 3\sqrt{16} & \quad 3\sqrt{49} & \quad 3\sqrt{144}
\end{align*}
\]
The most famous irrational number is \( \pi \) (pi).

\[ 3.141592654... \]

But the approximations for \( \pi \) such as 3.14 and \( \frac{22}{7} \) are both rational.

Examples

Which two are irrational?

\[ 3.14 \quad \frac{22}{7} \quad 3.14159... \quad \pi \]

Accurately place pi on the number line.
EXAMPLES

Accurately place the irrational expressions on the number line.

\[ \sqrt{18} \quad \frac{\sqrt{18}}{2} \]

\[ 2\pi \quad 2\pi - 1 \]
# PRACTICE

Copy the irrational number.

<table>
<thead>
<tr>
<th>$\sqrt{49}$</th>
<th>5.61</th>
<th>.7</th>
<th>1.732...</th>
<th>1.16</th>
<th>$\sqrt{60}$</th>
<th>$\frac{22}{11}$</th>
<th>$\sqrt{169}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{64}$</td>
<td>.555...</td>
<td>$\sqrt{75}$</td>
<td>$\frac{1}{4}$</td>
<td>3$\sqrt{25}$</td>
<td>3$\sqrt{36}$</td>
<td>3$\sqrt{42}$</td>
<td>3$\sqrt{49}$</td>
</tr>
</tbody>
</table>
PRACTICE

Accurately place the irrational expressions on the number line.

\[ \sqrt{41} \]  \hspace{1cm} \frac{\sqrt{41}}{2}
Accurately place the irrational expressions on the number line.

\[ \pi + 3.5 \]